profiling; F_0, area of the initially smooth surface, m^2; F_p, area of the profiled surface, m^2; C_f, local friction coefficient; St, Stanton number; Re^{**}, Re_T^{**}, Reynolds numbers over the thickness of momentum and energy loss.

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ANALOG OF THE RAYLEIGH EQUATION FOR THE PROBLEM OF BUBBLE DYNAMICS IN A TUBE

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UDC 536.248.2.001.5

Analysis is made of the growth of a vapor (gas) bubble in a horizontal tube filled with an ideal liquid.

As is well known [1], the dynamics of a gas (vapor) bubble in an unrestricted volume of an ideal liquid is described by the Rayleigh equation

$$\frac{\Delta P}{\rho} = \frac{P'' - P_\infty}{\rho} = \frac{3}{2}\dot{R}^2 + R\ddot{R}. \tag{1}$$

For the problem of bubble growth in a tube, vital for a number of applications connected with boiling, the "spherical" equation (1) is inapplicable in the general case. The present work gives the derivation of a "cylindrical" analog for the Rayleigh equation (1).

We consider first the case of expansion of a spherical bubble in the symmetry center of a horizontal tube with radius R_0 and length $2l$ (Fig. 1a). We obtain the pressure difference between the bubble $P''(t)$ and the liquid at the outlet from the tube P_∞ = const by integrating the z-projection of a momentum equation for z at r = 0 going from z = R (bubble surface) to z = l (outlet from the tube):

$$\frac{P'' - P_\infty}{\rho} = \frac{1}{2}(u_l^2 - u_R^2) + \int\limits_R^l \frac{\partial u}{\partial t}\, dz. \tag{2}$$

To calculate the right side of Eq. (2) we consider the velocity field from a point mass source at the axis of an infinite tube [2]. The velocity potential φ and the longitudinal velocity u at the axis z (for r = 0) of the flow are written in the form

$$\bar{\varphi} = \frac{\varphi}{u_\infty} = -\frac{1}{2\bar{z}} - \frac{1}{\pi}\int\limits_0^\infty \frac{K_1(\varepsilon)}{I_1(\varepsilon)}\cos(\varepsilon\bar{z})\, d\varepsilon, \tag{3}$$

Moscow Power Engineering Institute. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 63, No. 1, pp. 28-31, July, 1992. Original article submitted September 27, 1991.

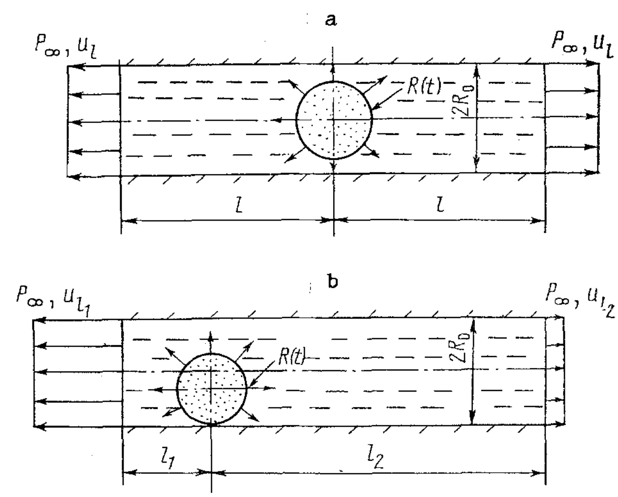

Fig. 1. Chart of gas (vapor) bubble growth in a tube: a) symmetric case; b) nonsymmetric case.

$$\bar{u} \equiv \frac{u}{u_\infty} = \frac{1}{2\bar{z}^2} + \frac{1}{\pi}\int_0^\infty \frac{K_1(\varepsilon)}{I_1(\varepsilon)} \varepsilon \sin(\varepsilon\bar{z})\, d\varepsilon. \tag{4}$$

Here $\bar{z} = z/R_0$ is the dimensionless longitudinal coordinate and u_∞ is the uniform flow velocity, as $\bar{z} \to \pm\infty$ (asymptotic source velocity); $I_1(\varepsilon)$ and $K_1(\varepsilon)$ are the modified Bessel functions of the first and second kind, respectively [3]. As $\bar{z} \to \infty$, from (3), (4) we have

$$\bar{\varphi} \approx -\frac{1}{2\bar{z}}, \quad \bar{u} \approx \frac{1}{2\bar{z}^2}.$$

To determine the asymptotic form of $\bar{\varphi}$, when $\bar{z} \to \pm\infty$, we will calculate from (3) the even derivatives of $\bar{\varphi}^{(2n)}(\bar{z})$ and add them termwise with the weight factors a_n so that, as a result, we can obtain in the right side the tabulated integral [4]:

$$\int_0^\infty \frac{K_1(\varepsilon)}{I_1(\varepsilon)} \cos(\varepsilon\bar{z}) \left(\sum_{n=0}^{\infty} (-1)^n a_n \varepsilon^{2n+1}\right) d\varepsilon = \frac{\pi}{2} \frac{\bar{z}}{(1+\bar{z}^2)^{1/2}}.$$

When $\bar{z} \to \infty$, we obtain the infinite-order uniform differential equation:

$$\sum_{n=0}^{\infty} a_n \bar{\varphi}^{(2n)} = 0, \tag{5}$$

having the solution

$$\bar{\varphi}(\bar{z}) \approx \sum_{n=1}^{\infty} b_n e^{-\alpha_n \bar{z}} \approx b_1 e^{-\alpha_1 \bar{z}}, \tag{6}$$

where α_n are the zeros of the Bessel function $J_1(\alpha_n)$, so $\alpha_1 = 3.832$. For estimates we use the interpolation relation for velocity potential, exact in asymptotic forms, as $\bar{z} \to 0$ and $\bar{z} \to \infty$:

$$\bar{\varphi} \approx -\frac{(\alpha_1/2)}{e^{\alpha_1 \bar{z}} - 1}. \tag{7}$$

Calculating the right side of the momentum equation (2) with (7) and estimating the terms for $R_0/l \ll 1$, we obtain

$$\frac{P'' - P_\infty}{\rho} = \dot{u}_l \left[l + \frac{1}{2}\left(\frac{R_0}{R}\right)^2 \right] + \frac{u_l^2}{2}\left[1 - \frac{1}{4}\left(\frac{R}{R_0}\right)^4 \right]. \tag{8}$$

The liquid velocity at the outlet from the tube u_l is connected with the bubble growth velocity $\dot{R} = dR/dt$ by the balance of source mass

$$u_l = \frac{2R^2\dot{R}}{R_0^2}. \tag{9}$$

The substitution of u_l from (9) into (8) gives the desired analog of the Rayleigh equation for the case of spherical bubble growth in the symmetry center of a tube filled with liquid:

$$\frac{P'' - P_\infty}{\rho} = \left(\frac{3}{2}\dot{R}^2 + R\ddot{R}\right) + \frac{2Rl}{R_0^2}(2\dot{R}^2 + R\ddot{R}). \tag{10}$$

For the case of constant pressure difference ($\Delta P = P'' - P_\infty = \text{const}$) Eq. (10) has the exact solution

$$R = \frac{3}{8}\frac{R_0^2}{l}\left\{\left[1 + \sqrt{\frac{2}{3}\frac{\Delta P}{\rho}}\left(\frac{4lt}{R_0^2}\right)\right]^{2/3} - 1\right\}. \tag{11}$$

For $Rl/R_0^2 \leq 1$ (initial stage of growth), from (11) the well-known "Rayleigh" law of growth [1] follows:

$$R = \sqrt{\frac{2}{3}\frac{\Delta P}{\rho}}\, t. \tag{12}$$

For sufficiently large values of the parameter $l/R >> 1$ ("long tubes") the "Rayleigh" stage of growth covers a negligible portion of the total time (until the whole section of the tube is filled with the bubble). The solution of (12) yields

$$R = \left(\frac{\Delta P}{\rho l}\right)^{1/3}\left(\frac{3}{4}R_0 t\right)^{2/3}. \tag{13}$$

Analyzing the solution (11) permits, for $Rl/R_0^2 \geq 1$ and $l/R_0 >> 1$, confinement to the following asymptotic form of Eq. (8):

$$\Delta P \approx \rho \dot{u} l. \tag{14}$$

Expression (14) permits generalization of the solution of (10) to the general case of bubble growth on the tube wall at different distances from the outlets $l_1 \neq l_2$ (Fig. 1b). Thus, instead of the symmetric case (8), we will have two equalities

$$\frac{\Delta P}{\rho} = l_1 \dot{u}_{l_1} = l_2 \dot{u}_{l_2}, \tag{15}$$

taking the source mass balance into account

$$u_{l_1} + u_{l_2} = \frac{2R^2\dot{R}}{R_0^2} \tag{16}$$

we obtain the "cylindrical" analog of the Rayleigh equation:

$$\frac{\Delta P}{\rho} = \frac{2Rl_*}{R_0^2}(2\dot{R}^2 + R\ddot{R}), \tag{17}$$

where $l_* = l_1 l_2/(l_1 + l_2)$. For $l_1 = l_2 = l$ we obtain the symmetric case (10), considered above.

The Rayleigh equation analog (17) may be used in calculating the dynamics of spherical bubbles in long tubes. We shall note that the above-developed approach permits us to take account of the influence of liquid viscosity and channel orientation on the bubble dynamics.

NOTATION

P'', pressure in bubble; P_∞, pressure at infinity; R, bubble radius; z, r, longitudinal and radial coordinates; t, time; φ, velocity potential; u_l, liquid velocity at the outlet from the tube; R_0, tube radius; l_1, l_2, distances from the bubble coordinates to the outlets from the tube.

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STABILITY OF NONISOTHERMAL FLOW OF A VISCOUS LIQUID IN AN ANNULAR CHANNEL

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UDC 532.516

In nonisothermal flow of a viscous liquid in an annular channel between coaxial cylinders where the outer cylinder has finite dimensions and is stationary, and the inner cylinder infinitely moves along the axis, the central position of the latter is unstable. When superimposing a thermal field, principally it is possible to create as large a force as required which holds the inner cylinder exactly on center.

Two coaxial cylinders with a liquid between them are considered. A similar system is often met with in various fields of engineering [1, 2]. In particular, they are a matter of practical interest in technology of polymer coatings when the inner infinite cylinder (fiber) moves in the direction of its axis and the outer cylinder of finite dimensions acts as a gauging device. A polymer solution is fed to the gauge inlet, and at the outlet this solution is entrained by the fiber forming a polymer coating on it. The important parameters of such a process are the thickness of a polymer film on the fiber and its uniformity.

In this problem the gap between the inner and outer cylinders may act as a perturbation which makes it possible to obtain, by approximate methods, analytical expressions for characterizing the system.

1. We consider the general case of nonisothermal stationary nonaxisymmetric motion of a viscous liquid in an annular channel between cylinders when the outer one has a cone shape. To simplify the problem, we make the usual assumptions of developed flow [3, 4]. The liquid viscosity is dependent on temperature [5]. In the final analysis we consider the system of equations

$$\frac{\partial P}{\partial r} = 0; \tag{1}$$

$$\frac{\partial P}{\partial z} = \mu_0 \left(\frac{\partial^2 V_z}{\partial r^2} + \frac{\partial V_z}{r \partial r} - \frac{k}{T_0} \frac{\partial T}{\partial r} \frac{\partial V_z}{\partial r} \right); \tag{2}$$

$$\frac{\partial P}{r \partial \varphi} = \mu_0 \left(\frac{\partial^2 V_\varphi}{\partial r^2} + \frac{\partial V_\varphi}{r \partial r} - \frac{V_\varphi}{r^2} - \frac{k}{T_0} \frac{\partial T}{\partial r} \left(\frac{\partial V_\varphi}{\partial r} - \frac{V_\varphi}{r} \right) \right); \tag{3}$$

$$\frac{\partial V_z}{\partial z} + \frac{\partial V_r}{\partial r} + \frac{V_r}{r} + \frac{\partial V_\varphi}{r \partial \varphi} = 0; \tag{4}$$

$$c \left(V_z \frac{\partial T}{\partial z} + V_r \frac{\partial T}{\partial r} \right) = \lambda \left(\frac{\partial^2 T}{\partial r^2} + \frac{\partial T}{r \partial r} \right) \tag{5}$$

Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 63, No. 1, pp. 32-37, July, 1992. Original article submitted August 20, 1991.

1062-0125/92/6301-0675$12.50